spot price  
: strike price  
: time to maturity,   
 risk-free rate   
 dividend yield  
: annualized volatility (standard deviation) of an underlying asset   
are constants under BS framework, if deterministic, notate   
: implied volatility calculated by the Black-Scholes formula given and

: forward price ( under Black-Scholes framework)  
: log forward moneyness  
: total variance

**SVI model**

Parameters to calibrate:   
- : vertical shift, slope of wings, : clockwise rotation, : horizontal shift, : curvature  
Log-forward moneyness: , Total implied variance:

No Arbitrage SVI Algorithm

Input: (log-forward moneyness), (total implied variance)  
Object: find SVI parameters under no-arbitrage conditions

1. Choose and s.t.
2. Compute numerically and parametrize for positive
3. Compute numerically

Mathematical Backgrounds

1. Local volatility
2. Butterfly arbitrage
3. Calendar arbitrage
4. Durrleman condition
5. Lee’s moment formula (limits of slope on both wingtips – large and small strike tails)
6. Convergence of Heston model to SVI

No-butterfly arbitrage conditions

1. **Fukasawa (2012)** – necessary condition to prevent Butterfly arbitrage   
   No butterfly arbitrage are strictly increasing
2. **Normalizing SVI parameters**  
   Define , ,   
    where (and , )  
   In this case, we can reduce the number of parameters ( and seperate and as a variable
3. **Durrleman condition**  
    where and   
    No butterfly arbitrage
4. **Classifying cases**(A1): , , fundament case – SVI well defined & positive total variance (when singularity problem occurs)  
   (A2):   
   (B1): ,   
   (B2):,   
   (B3):,   
   (B4):,
5. Investigating Fukasawa necessary no arbitrage conditions
   1. Lemma 5.1. (Limits of )  
       and
6. (the range of )  
   Find the interval of that satisfies the Fukaswa condition  
    where and is the minimizer of (typically,
   1. Special case:   
      Always exists the interval since and   
       If we properly select in , for .
   2. General case   
      In this case, it might not be possible to determine the interval (possible )

Normalizing SVI parameters

, ,

Fukasawa conditions Given , get conditions on , conditions on

5.2. The conditions as an interval for

*Not every choice allows , especially when*

Thus, we need to find proper parameters that allow to exist the interval for :

Let’s find the minimum point of and

5.2.2. Computation of the interval for under (B1):

Assume , (A1), and (B1)

Proposition 5.5.

Let

Corollary 5.6.

There is a unique such that   
The interval is non-empty   
In this case, the distance increases with

A screenshot of a computer

Description automatically generated

Notations

Log-forward moneyness

SVI model: